

## Standardized Chargeability Calculations in Titan 24 IP Measurements\*

Applies To: Titan 24 DC/IP

### **Introduction**

This document describes Quantec's approach to standardizing chargeability calculations in Titan 24 operations. The term chargeability implies that our preferred approach to calculating IP is via time domain (half-duty square-wave) responses. We adopt several criteria in this regard:

- a) IP response magnitudes should not change as the charge and decay averaging (integration) windows change.
- b) Units and amplitude should be generally familiar to the mining geophysics community; i.e. it should be straight forward for most to compare IP anomalies with other background and anomalous responses in their experience. However, no consideration is given to the early Newmont  $M_{331}$  standard whose units were milliseconds.

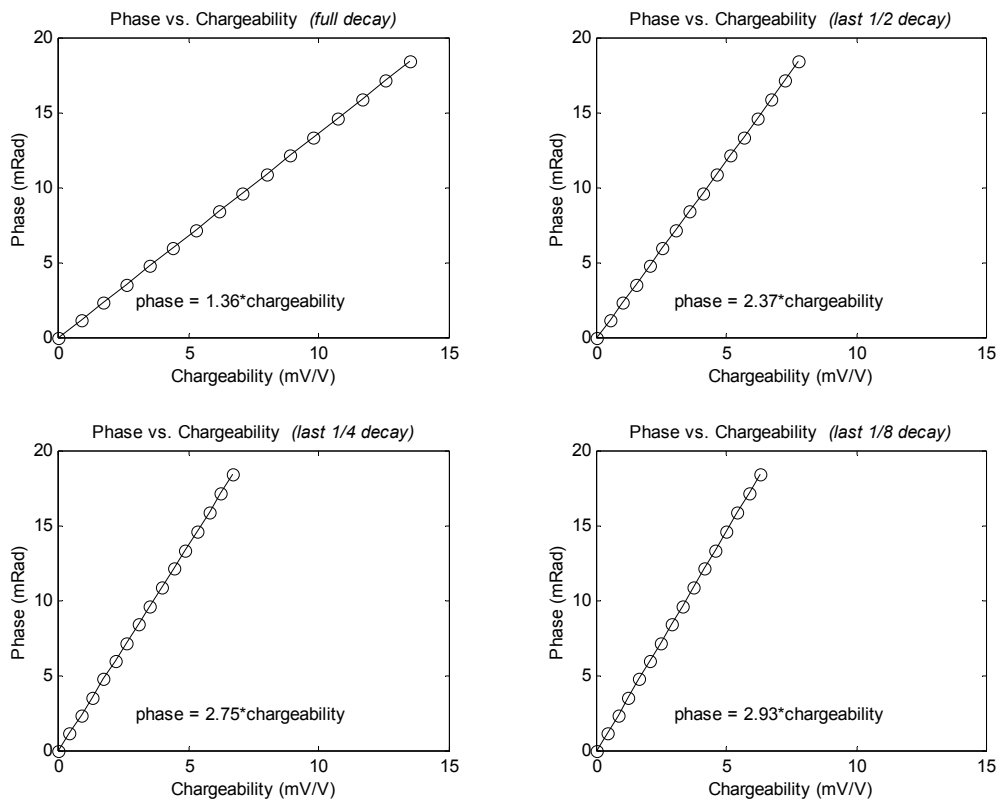
### **Phase vs. Chargeability**

Because of EM coupling considerations we strongly prefer calculating the IP effect magnitude in the time-domain, based on a half-duty square-wave response. However, we note that there is a direct (proportional) relationship between chargeability and phase, regardless of the averaging (integration) window in the decay curve. This is illustrated in Figure 1 below for various averaging windows. It is important to recognize that in mentioning averaging or integration window in the decay and charge curves, we specifically mean calculating the average voltages in those windows (thereby normalizing to integration time limits) and not simply integrating the decay as implied in the early Newmont approach. It should be apparent that this provides a much greater degree of scale uniformity for varying integration window sizes. Hence time-domain chargeability units are mV/V, not mSec.

It must be further explained that in the chargeability values forming the ordinates in the graphs below, the averaging windows represent the same relative locations in both the charge and decay curves. The advantages of wider averaging windows, providing they do not span across earlier-time EM coupling, apply to both average charge and decay voltage estimates.

The reader should bear in mind that the chargeabilities are not strongly affected by the location and width of the charge curve averaging window, but are somewhat sensitive to the decay averaging window. Hence, even if one adopts the more common standard of using the very tail of the charge curve for calculating  $V_p$ , as opposed to the standard used this memo, the chargeability values are minimally affected and are still proportional to phase.

\*From: Kingman, J.E.E (2008, November). *Re: Quantec Geoscience chargeability scale/units standards* (internal email report).



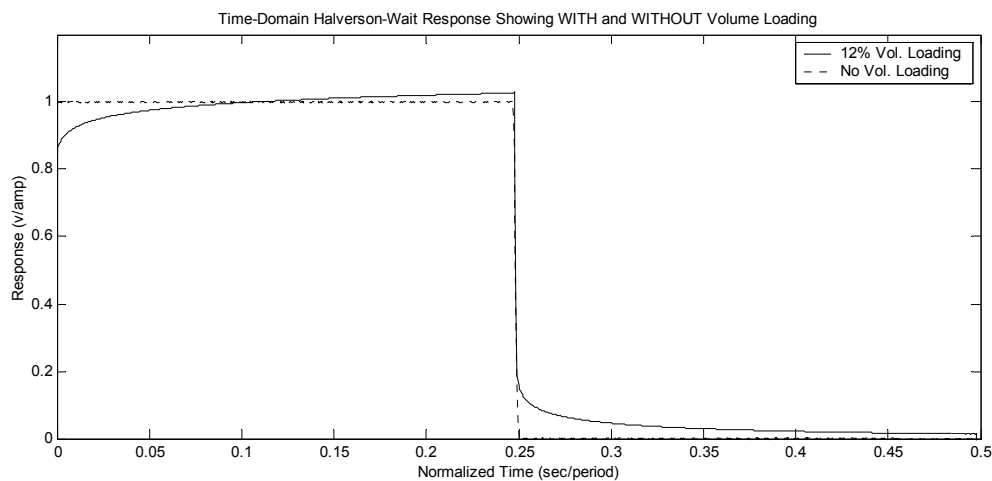
**Figure 1 - illustration of the proportional relationship between phase (mRad) and chargeability (mV/V) for various charge/decay averaging windows**

Given the strongly linear (proportional) relationship between phase and chargeability, it is clear that for a chosen averaging window one can reasonably convert chargeability to phase by multiplying by the appropriate calibration constant. In view of the criteria mentioned in the introduction, especially as far as choosing a scale and units standard that is familiar, we prefer to present chargeability or IP effect values in terms of milliradians, but estimate those values in the time domain<sup>1</sup> through chargeability calculations. Bear in mind that even if we prefer chargeability units, there is still the need to normalize to some window or time gate standard for varying integration limits. Choosing phase instead of this standard integration range simply provides a more familiar standard. Furthermore, it allows us a standard that does not require defining the filtering level; i.e. if you establish a time-domain standard, you must also establish a limited bandwidth standard.

The direct relationship between phase and chargeability is easy to understand when considering that the chargeability calculation (decay voltage/charge voltage) is directly related to the tangent of the phase. In other words, the addition of polarizable particles to an otherwise non-polarizable media such as in the Halverson-Wait model, causes a quadrature or out-of-phase response component which is the decay and slightly altered charge curves in the half-duty square-wave response.

<sup>1</sup> This approach harkens back to the Anaconda approach and nomenclature of 'area based phase'. Anaconda chose units of minutes instead of milliradians

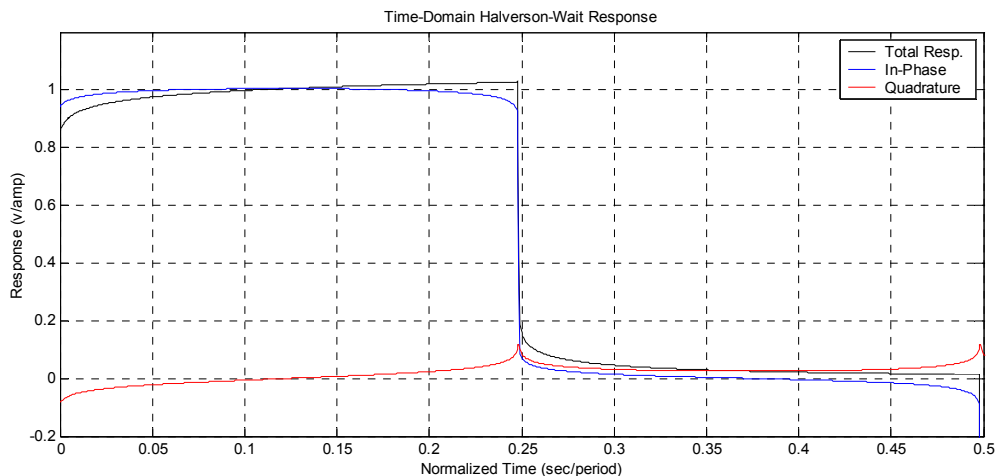
Figure 2 helps to illustrate this. A non-chargeable time-domain<sup>2</sup> response has a perfectly flat charge or on-time response and no decay when ignoring EM coupling. If we consider the Halverson-Wait model, adding a 12% volume loading of polarizable spheres (conductive with complex impedance coating) to a non-polarizable media causes an initial (short-lived) drop in the bulk resistivity as compared with the media itself, followed by an increase in bulk resistivity as the spheres 'charge up'.



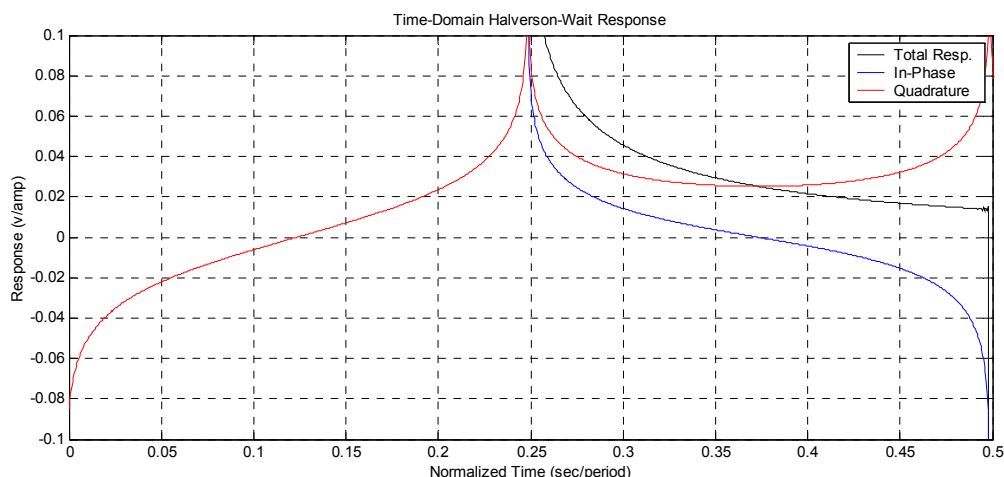
**Figure 2 - polarizable and non-polarizable time-domain responses**

<sup>2</sup> Half-duty square-wave presumed

In Figure 3 the response curve is broken in to its in-phase and quadrature components. We know that the quadrature component, by virtue of its even symmetry, must cross the zero amplitude mark at the mid-point of the charge curve. Similarly, we know that the in-phase component must cross the zero amplitude mark at the mid-point of the decay. This is better illustrated in Figure 4, whose vertical range is smaller than in Figure 3.



**Figure 3 - polarizable (Halverson-Wait) response separated in to quadrature and in-phase**



**Figure 4 - enlargement of the in-phase and quadrature portions of the Halverson-Wait response**

There is an interesting phenomenon concerning the in-phase or charging response. When we add a volume loading of conductive spheres, the resistivity at high frequencies of the bulk material should drop. However, at sufficiently low frequencies or long enough charge times, the spheres become resistive and so the bulk resistivity increases. Therefore, we find a somewhat complicated, frequency dependent relationship in comparing the resistivities of a Halverson-Wait volume loading with that of the barren host matrix.

It is easy to see that for average voltages near the center of the charge and decay periods, the  $V_p$  is dictated only by the in-phase component, and similarly the average decay voltage is dictated only by the quadrature

component. But what about when the averaging window or time-gate is not centered in the charge/decay curves? As we shift the averaging window within the charge and decay curves, we are in essence, picking up small portions of the 'other' components (quadrature or in-phase). In other words we have the following:

$$V_p(t_0) = \alpha_p(t_0) InP + \beta_p(t_0) Q \quad (1.1)$$

And

$$V_s(t_0) = \alpha_s(t_0) InP + \beta_s(t_0) Q \quad (1.2)$$

Where:

$V_p(t_0)$	- primary (time-domain) averaged charge voltage (normalized to current)
$V_s(t_0)$	- secondary (time-domain) averaged decay voltage (normalized to current)
$InP$	- in-phase voltage at the fundamental (normalized to current)
$Q$	- quadrature voltage at the fundamental (normalized to current)
$\alpha(t_0), \beta(t_0)$	- proportionality constants
$t_0$	- delay time for time-domain average charge and decay voltage calculations

As the averaging window (or  $t_0$ ) moves, the proportionality constants ( $\alpha$ ,  $\beta$ ) change, but otherwise the relationship is exact for all volume loadings (chargeabilities). For  $t_0 \approx 0$ ,  $\beta_p$  and  $\alpha_s$  are close to zero, regardless of the spectral character of the response. However, let us consider large  $t_0$  (near the last 1/8<sup>th</sup> of the decay, and a very small time constant response such that the tail end of the decay curve is very close to zero. In this case we would calculate a very small chargeability, and for the same reason,  $\alpha_s(t_0) \approx \beta_s(t_0)$ . Hence, we may falsely indicate a small phase simply because the time constant is small and  $t_0$  is large. These are inescapable pitfalls of trying to circumvent EM coupling while running narrow-band IP.

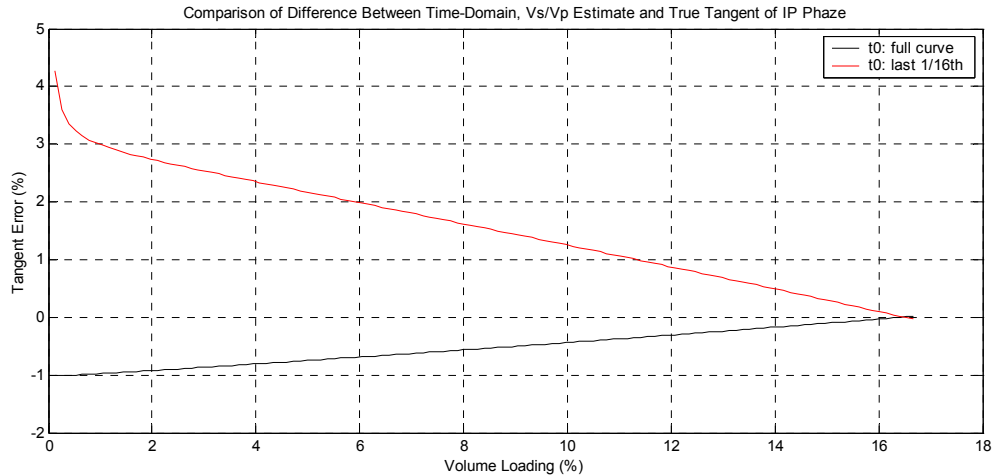
We find as a practical matter, that:

$$\frac{V_s(t_0)}{V_p(t_0)} = \frac{\alpha_s(t_0) IPz + \beta_s(t_0) Q}{\alpha_p(t_0) IPz + \beta_p(t_0) Q} \quad (1.3)$$

... can be approximated by:

$$\frac{V_s(t_0)}{V_p(t_0)} \approx \frac{\beta_s(t_0) Q}{\alpha_p(t_0) InP} = k \frac{Q}{InP} \quad (1.4)$$

The approximation of Equation 1.4 serves reasonably well, regardless of the size of the time-domain averaging window, as shown in Figure 5 below:



**Figure 5 - errors resulting from the approximated proportional relationship between time-domain  $V_s/V_p$  and the actual tangent of the IP phase at the fundamental.**

Note that Figure 5 simply confirms what we knew to be true on the basis of the results displayed in Figure 1. If we use the full expression of Equation 1.3, then the relationship between the time-domain chargeability  $V_s/V_p$  and the true tangent at the fundamental, is exact under the presumption of standard spectral values ( $rValue = 1.0$ ,  $kValue = 0.2$ ).

Equation 1.3 is transformable to the following:

$$\begin{aligned}
 \frac{Q}{IPz} &= \frac{a_s(t_0)V_p(t_0) + b_s(t_0)V_s(t_0)}{a_p(t_0)V_p(t_0) + b_p(t_0)V_s(t_0)} \\
 &= \frac{\begin{bmatrix} V_p & V_s \end{bmatrix} \begin{bmatrix} a_s \\ b_s \end{bmatrix}}{\begin{bmatrix} V_p & V_s \end{bmatrix} \begin{bmatrix} a_p \\ b_p \end{bmatrix}} \quad (1.5)
 \end{aligned}$$

Given this model, if the proportionality constants  $a_s$ ,  $a_p$ ,  $b_s$ , and  $b_p$  are determined using  $t_0$  and Halverson-Wait time and frequency domain results for standard spectral values, then the estimate of the tangent of the phase based on  $V_s/V_p$  is essentially exact. In many regards, it would make more sense to standardize IP units/scale as the tangent of the phase or the quadrature at the fundamental divided by the in-phase at the fundamental, whose units (like time-domain chargeability) would be volts per volt (or millivolts per volt). However, the scale and meaning of milliradians of phase are well known to the industry and so may serve as a better normalization standard. Regardless, for IP effects less than 200 milliradians, the phase and the tangent are essentially the same – so we may refer to Quantec normalized IP results either as phase (milliradians) or the tangent of the phase (millivolts/volt), and reasonably claim that both scales/values are the same.

## Quantec IP Standard

We have two primary issues at hand regarding our scale standard for IP responses: 1) our standard for Titan 24 (and future proprietary systems), and 2) converting results from commercial IP equipment (Zonge, IRIS, Scintrex) to our standard. An equally important issue is that of converting our IP units to PFE for modeling and inversion purposes. This latter issue is addressed in the next section. First we discuss Quantec's IP scaling standard in the context of the Titan 24 and commercial systems.

### Titan 24 System

As mentioned, we prefer to estimate IP responses using a (time-domain) half-duty square-wave excitation standard, but convert those chargeability results to units of phase. Our specific procedure and algorithm is as follows:

1. Determine the earliest time for which EM coupling has died out sufficiently. This time is called the averaging or integration *start time* or  $t_0$ .
2. Determine the latest charge/decay time that is minimally affected by sigma-delta and low-pass (usually Hanning window moving average) filtering - called the averaging or integration *end time* or  $t_e$ . For example, if a 15 tap Hanning window moving average filter has been applied to remove powerline and high frequency noise, then the last charge/decay sample should be 15 samples back (earlier) from the excitation switch.
3. Adjust the *start time* ( $t_0$ ) so that  $t_e - t_0$  (equated to number of samples) exactly spans an integer number of power-line signal periods. Clearly, this can only be done for transmitted (fundamental) frequencies that are much lower than the power-line frequency.
4. Using the charge and decay sample numbers that equate to the averaging window<sup>3</sup> defined by  $t_0$  and  $t_e$ , calculate the average charge and decay voltages. This average may involve a non-uniform weighting to further improve rejection of power-line noise.
5. Calculate the theoretical Halverson-Wait half-duty time-domain response using identical filtering to that applied to the measured data response estimate, and presuming the following spectral parameters:

<i>volume loading</i> -	<i>0.125 (this value is unimportant)</i>
<i>rValue</i> -	<i>1.0</i>
<i>kValue</i> -	<i>0.2</i>

6. For the standard Halverson-Wait spectral parameters mentioned, the synthesized time-domain response and the  $t_0 - t_e$  averaging window, calculate the transform coefficients in Equation 1.5 relating theoretical tangent of the phase at the excitation fundamental to chargeability for the selected averaging window.
7. Using the transform coefficients calculated in step 6, convert all estimated/measured charge and decay voltages (using the specified averaging window) to the tangent of the phase (millivolts/volt) and then to phase (milliradians).

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<sup>3</sup> In practice this averaging window is tapered slightly to widen the stop-band notches and thereby provide enhanced power-line noise rejection.

This is the algorithm used in the Titan 24 data processing. The relationship between Titan 24 chargeability units and other frequency domain systems is straightforward – Quantec’s time-domain based phase equates to frequency domain based phase, except that it (Quantec’s) enjoys superior EM decoupling. The question remains, however, as to how to relate Titan 24 phase results to those of other systems such as the GDP-32 (Zonge) or Elrec-10 (Iris).

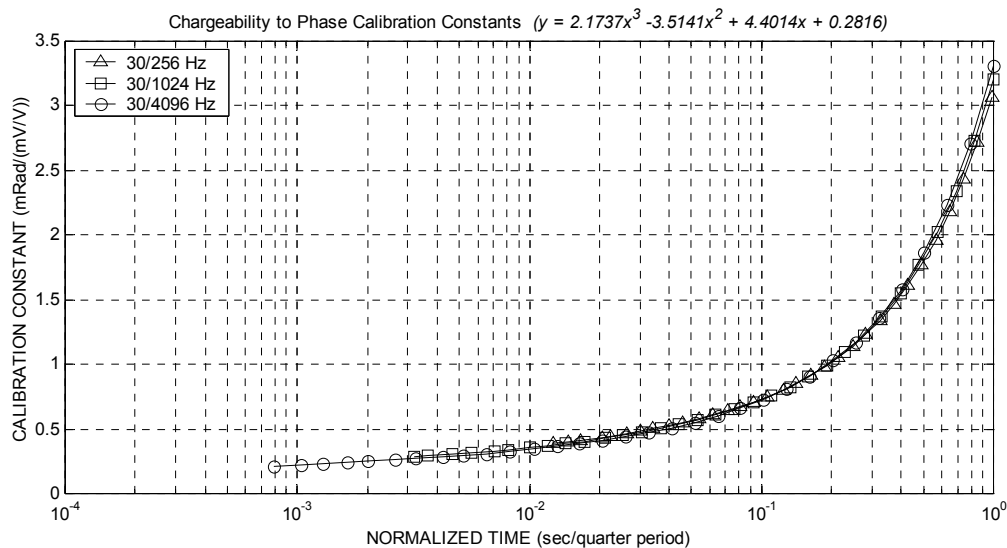
### Commercial IP Systems and Results

For time-domain systems with a series of decay windows, we see the following relationships<sup>4</sup>, illustrated in Figure 5 below. In this illustration the primary voltage in the chargeability calculations is taken near the end of the charge curve. Note the agreement between the various pulse-lengths when time scale is normalized. A good approximation relating phase to chargeability for all these curves is:

$$\phi = m(2.1737t^3 - 3.5141t^2 + 4.4014t + 0.2816) \quad (1.6)$$

Units:

$\phi$	milliradians
$m$	mV/V
$t$	sec/sec (decay time normalized to pulse-length)



**Figure 6 - Ratio between phase and IP decay times, when time is normalized to the pulse length; the relationship is virtually identical for all pulse lengths**

<sup>4</sup> These presume essentially infinite bandwidth in the time-domain response. In fact it is difficult to discern bandwidth specifications in commercial IP receivers, but the presumption of infinite bandwidth seems a reasonable guess based on personal investigations by the author.

Hence, the reader may use this formula or Figure 5 to determine the proportionality constant relating phase to time-domain chargeability time-gates.

### Chargeability Units in IP Modeling and Inversion

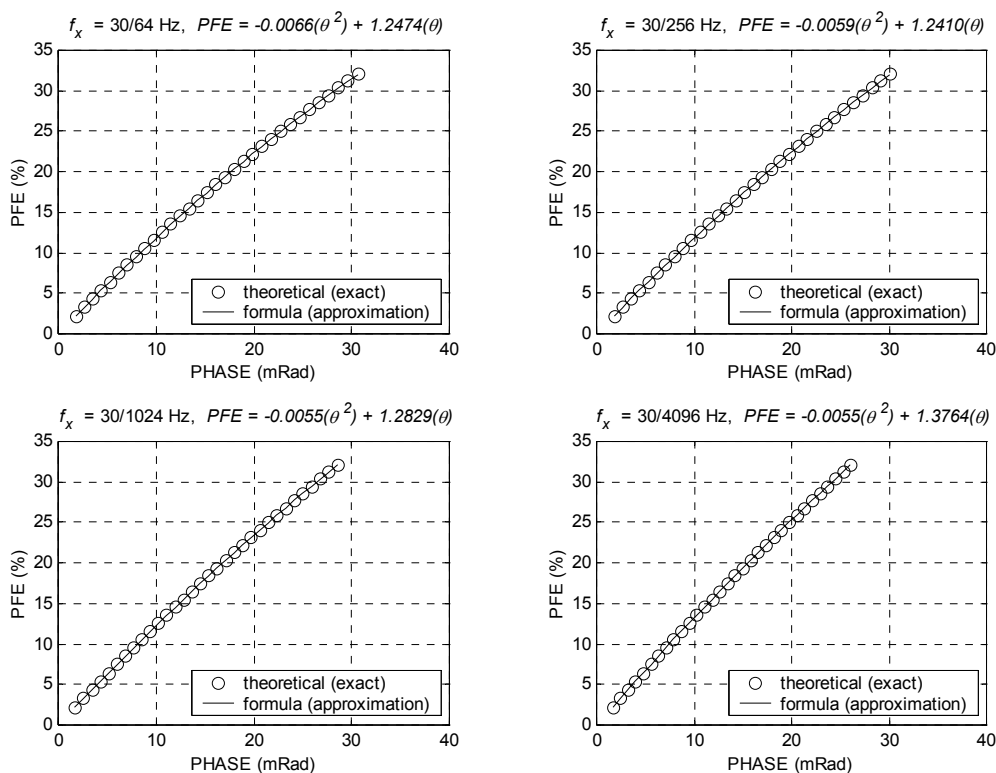
Most IP modeling programs work by running two models whose resistivities reflect differences related to Percent Frequency Effect (PFE<sup>5</sup>). For a given spectral character, the indicated chargeability or phase may not necessarily reflect the true PFE of the media. This is because the measured, EM coupling-free frequency band is rarely broad enough to indicate the peak phase or broadband PFE. There are a number of issues in this regard related to modeling spectral IP behavior. In this memo we limit the discussion to non-spectral modeling only. In the Halverson-Wait model the theoretical PFE (for infinite bandwidth), which equates to the theoretical chargeability in the Cole-Cole equation, is defined by the volume loading:

$$\frac{PFE_0^\infty}{100} = m_0 = \frac{9\nu}{2 + 3\nu} \quad (1.7)$$

As mentioned, it is in general impossible to know the volume loading unless the *rValue* and *kValue* are also known. The recommended standard in lieu of that is to presume standard (typical) Halverson-Wait parameters: *rValue* = 1.0 and *kValue* = 0.2, as we do in calculating time-domain based phase. The next step is to relate volume loading to our measured average charge and decay voltages,  $V_p(t_0)$  and  $V_s(t_0)$ , under the presumption of standard Halverson-Wait parameters. The relationship between phase and PFE for various pulse lengths and standard Halverson-Wait parameters is illustrated in Figure 6 below.

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<sup>5</sup> We presume the classical definition of PFE as  $100 \left( \frac{\rho_0 - \rho_\infty}{\rho_0} \right)$



**Figure 7 - Phase vs. PFE for various pulse lengths and presuming standard Halverson-Wait spectral parameters (r-value = 1.0 and k-value = 0.2).**

Note that these are not quite linear curves. The quadratic formulae in the titles track the theoretical decays very closely. There is probably little merit to carrying this relationship to extreme precision. In fact, there are good arguments for simplifying to a straight proportional relationship. We offer the following two options based on a transmitter frequency of 30/256 Hz:

$$PFE = -0.0059\theta^2 + 1.2410\theta$$

$$\theta = \frac{-1.2410 + \sqrt{1.5401 - 0.02350PFE}}{-0.01175}$$

... or (approximately)

$$PFE = 1.0545\theta \tag{1.8}$$